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Lecture (7)

Transmission line Models
&
Calculation

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Transmission line models

- Transmission line models parameters include:

series resistance & inductance & shunt capacitance.

- In this lecture we shall discuss the various models of the line.
- The line models are classified by their length. These classifications are:

1. Short line approximation for lines that are less than 80 km long.
2. Medium line approximation for lines whose lengths are between 80 km to 240 km.
3. Long line model for lines that are longer than 240 km.

ABCD Parameters

- Consider the power system In this the sending and receiving end voltages are denoted by V_S and V_R respectively.
- Also the currents I_S and I_R are entering and leaving the network respectively.
- The sending end voltage and current are then defined in terms of the ABCD parameters as

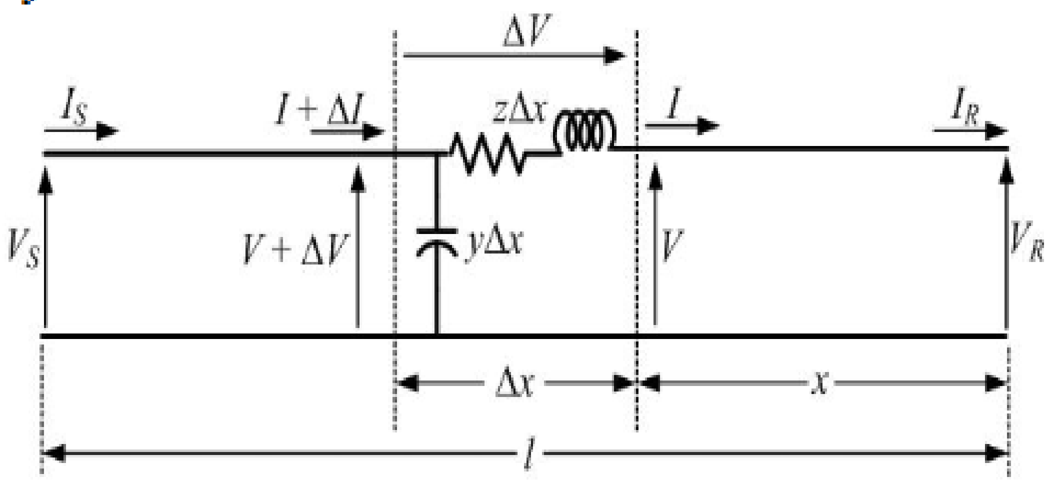
$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

1. Long line model

- For accurate modeling of the transmission line we must not assume that the parameters are lumped but are distributed throughout line.
- The single-line diagram of a long transmission line is shown The length of the line is l.
- consider a small strip Δx that is at a distance x from the receiving end.
- The voltage and current at the end of the strip are V and I respectively and the beginning of the strip are $V + \Delta V$ and $I + \Delta I$ respectively.

Cont.

- The voltage drop across the strip is then ΔV. Since the length of the strip is Δx, the series impedance and shunt admittance are $z\Delta x$ and $y\Delta x$. It is to be noted here that the total impedance and admittance of the line are $Z = z \times l$ and $Y = y \times l$



Long transmission line representation.

Cont.

From the circuit

$$\Delta V = Iz\,\Delta x \Rightarrow \frac{\Delta V}{\Delta x} = Iz \qquad (2.25)$$

Again as $\Delta x \to 0$, from (2.25) we get

$$\frac{dV}{dx} = Iz \qquad (2.26)$$

Now for the current through the strip, applying KCL we get

$$\Delta I = (V + \Delta V)y\,\Delta x = Vy\,\Delta x + \Delta Vy\,\Delta x \qquad (2.27)$$

The second term of the above equation is the product of two small quantities and therefore can be neglected. For $\Delta x \to 0$ we then have

$$\frac{dI}{dx} = Vy \qquad (2.28)$$

Taking derivative with respect to x of both sides of (2.26) we get

$$\frac{d}{dx}\left(\frac{dV}{dx}\right) = z\frac{dI}{dx}$$

Substitution of (2.28) in the above equation results

$$\frac{d^2V}{dx^2} - yzV = 0 \tag{2.29}$$

The roots of the above equation are located at $\pm\sqrt{(yz)}$. Hence the solution of (2.29) is of the form

$$V = A_1e^{x\sqrt{yz}} + A_2e^{-x\sqrt{yz}} \tag{2.30}$$

Taking derivative of (2.30) with respect to x we get

$$\frac{dV}{dx} = A_1\sqrt{yz}\,e^{x\sqrt{yz}} - A_2\sqrt{yz}\,e^{-x\sqrt{yz}} \tag{2.31}$$

Combining (2.26) with (2.31) we have

$$I = \frac{1}{z}\left(\frac{dV}{dx}\right) = \frac{A_1}{\sqrt{z/y}}e^{x\sqrt{yz}} - \frac{A_2}{\sqrt{z/y}}e^{-x\sqrt{yz}} \tag{2.32}$$

Let us define the following two quantities

$$Z_C = \sqrt{\frac{z}{y}}\ \Omega \quad \text{which is called the } \textbf{\textit{characteristic impedance}} \tag{2.33}$$

$$\gamma = \sqrt{yz} \text{ which is called the } \textbf{\textit{propagation constant}} \tag{2.34}$$

Then (2.30) and (2.32) can be written in terms of the characteristic impedance and propagation constant as

$$V = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \tag{2.35}$$

$$I = \frac{A_1}{Z_C} e^{\gamma x} - \frac{A_2}{Z_C} e^{-\gamma x} \tag{2.36}$$

Let us assume that $x = 0$. Then $V = V_R$ and $I = I_R$. From (2.35) and (2.36) we then get

$$V_R = A_1 + A_2 \tag{2.37}$$

$$I_R = \frac{A_1}{Z_C} - \frac{A_2}{Z_C} \tag{2.38}$$

Solving (2.37) and (2.38) we get the following values for A_1 and A_2.

$$A_1 = \frac{V_R + Z_C I_R}{2} \text{ and } A_2 = \frac{V_R - Z_C I_R}{2}$$

Also note that for $l = x$ we have $V = V_S$ and $I = I_S$. Therefore replacing x by l and substituting the values of A_1 and A_2 in (2.35) and (2.36) we get

$$V_S = \frac{V_R + Z_C I_R}{2} e^{\gamma l} + \frac{V_R - Z_C I_R}{2} e^{-\gamma l} \tag{2.39}$$

$$I_S = \frac{V_R / Z_C + I_R}{2} e^{\gamma l} - \frac{V_R / Z_C - I_R}{2} e^{-\gamma l} \tag{2.40}$$

Noting that

$$\frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l \quad \text{and} \quad \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l$$

We can rewrite (2.39) and (2.40) as

$$V_S = V_R \cosh \gamma l + Z_C I_R \sinh \gamma l \tag{2.41}$$

$$I_S = V_R \frac{\sinh \gamma l}{Z_C} + I_R \cosh \gamma l \tag{2.42}$$

The ABCD parameters of the long transmission line can then be written as

$$A = D = \cosh \gamma l$$

$$B = Z_C \sinh \gamma l \ \Omega$$

$$C = \frac{\sinh \gamma l}{Z_C} \text{ mho}$$

2. Medium transmission lines

- Medium transmission lines are modeled with lumped shunt admittance.

- There are two different representations depending on the nature of the network:

2.1 Nominal (π).

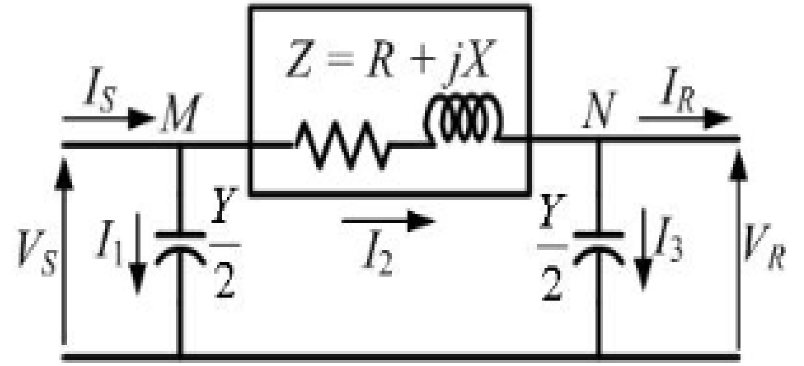
2.2 Nominal (T).

2.1 Nominal (π)

- in this representation the lumped series impedance is placed in the middle while the shunt admittance is divided into two equal parts and placed at the two ends.
- The nominal (π) representation is used for load flow studies.
- Also a long transmission line can be modeled as an equivalent (π) network for load flow studies.

- Let us define three currents I_1, I_2 and I_3 as indicated in Figure.
- Applying KCL at nodes M and N we get

$$I_s = I_1 + I_2 = I_1 + I_3 + I_R$$
$$= \frac{Y}{2}V_s + \frac{Y}{2}V_R + I_R$$



Nominal-π representation.

Again

$$V_s = ZI_2 + V_R = Z\left(V_R\frac{Y}{2} + I_R\right) + V_R$$
$$= \left(\frac{YZ}{2} + 1\right)V_R + ZI_R$$

Substituting

$$I_s = \frac{Y}{2}\left[\left(\frac{YZ}{2} + 1\right)V_R + ZI_R\right] + \frac{Y}{2}V_R + I_R$$
$$= Y\left(\frac{YZ}{4} + 1\right)V_R + \left(\frac{YZ}{2} + 1\right)I_R$$

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- the ABCD parameters of the nominal (π) representation

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\ \Omega$$

$$C = Y\left(\frac{YZ}{4} + 1\right)\text{mho}$$

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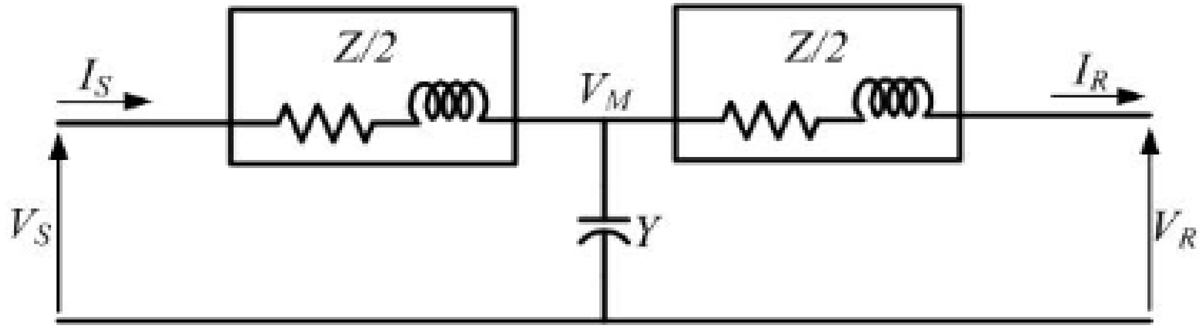
2.2 Nominal (T)

- In this representation the shunt admittance is placed in the middle and the series impedance is divided into two equal parts and these parts are placed on either side of the shunt admittance.



Nominal-T representation.

$$\frac{V_S - V_M}{Z/2} = YV_M + \frac{V_M - V_R}{Z/2}$$

Rearranging the above equation can be written as

$$V_M = \frac{2}{YZ+4}(V_S + V_R) \tag{2.16}$$

Now the receiving end current is given by

$$I_R = \frac{V_M - V_R}{Z/2} \tag{2.17}$$

Substituting the value of V_M from (2.16) in (2.17) and rearranging we get

$$V_s = \left(\frac{YZ}{2}+1\right)V_R + Z\left(\frac{YZ}{4}+1\right)I_R \tag{2.18}$$

Furthermore the sending end current is

$$I_S = YV_M + I_R \tag{2.19}$$

Then substituting the value of V_M from (2.16) in (2.19) and solving

$$I_R = YV_R + \left(\frac{YZ}{2}+1\right)I_R \tag{2.20}$$

Then the ABCD parameters of the T-network are

$$A = D = \left(\frac{YZ}{2} + 1 \right)$$

$$B = Z\left(\frac{YZ}{4} + 1 \right)\Omega$$

$$C = Y \text{ mho}$$

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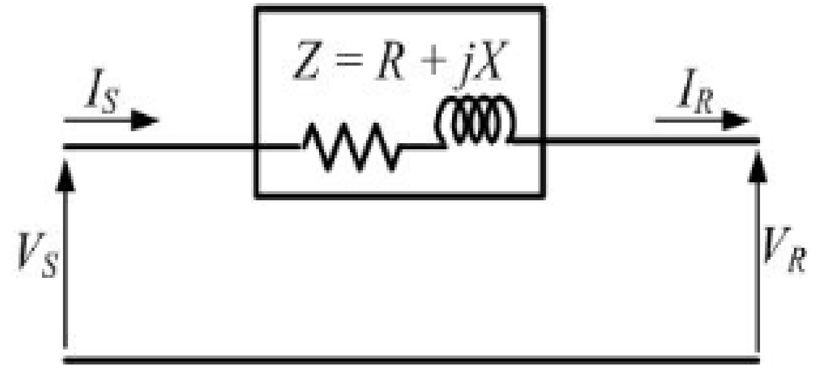
3. Short line approximation

- The shunt capacitance for a short line is almost negligible.
- The series impedance is assumed to be lumped as shown in Figure. If the impedance per km for an l km long line is $z_0 = r + jx$, then the total impedance of the line is: .

$$Z = R + jX = lr + jlx.$$



Short transmission line representation.

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Cont.

- The sending end voltage and current for this approximation are given by:

$$V_S = V_R + ZI_R$$

$$I_S = I_R$$

Therefore the ABCD parameters are given by

$$A = D = 1,\ B = Z\,\Omega \text{ and } C = 0$$

Transmission line Models
&
Calculation
Practical Solved Examples

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1. Long line model

The ABCD parameters of the long transmission line can then be written as

$$A = D = \cosh \gamma l$$

$$B = Z_C \sinh \gamma l \;\; \Omega$$

$$C = \frac{\sinh \gamma l}{Z_C} \text{ mho}$$

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2. Medium transmission lines

- Medium transmission lines are modeled with lumped shunt admittance.

- There are two different representations depending on the nature of the network:

2.1 Nominal (π).

2.2 Nominal (T).

2.1 Nominal (π)

- The ABCD parameters of the nominal (π) representation

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\ \Omega$$

$$C = Y\left(\frac{YZ}{4} + 1\right)\ \text{mho}$$

2.2 Nominal (T)

- The ABCD parameters of the nominal (T) representation

$$A = D = \left(\frac{YZ}{2} + 1\right)$$

$$B = Z\left(\frac{YZ}{4} + 1\right)\Omega$$

$$C = Y \text{ mho}$$

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3. Short line approximation

- The ABCD parameters of the short line representation

$$A = D = 1, B = Z\,\Omega \text{ and } C = 0$$

Example (1):

Example ⇨ Consider a 500 km long line for which the per kilometer line impedance and admittance are given respectively by $z = 0.1 + j0.5145\ \Omega$ and $y = j3.1734 \times 10^{-6}$ mho. Therefore

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.1 + j0.5145}{j3.1734 \times 10^{-6}}} = \sqrt{\frac{0.5241\angle 79^\circ}{3.1734 \times 10^{-6}\angle 90^\circ}} = \sqrt{\frac{0.5241}{3.1734 \times 10^{-6}}}\angle\left(\frac{79^\circ - 90^\circ}{2}\right)$$

$$= 406.4024\angle -5.5^\circ\ \Omega$$

and

$$\gamma l = \sqrt{yz} \times l = \sqrt{0.5241 \times 3.1734 \times 10^{-6}} \times 500\angle\left(\frac{79^\circ + 90^\circ}{2}\right)$$

$$= 0.6448\angle 84.5^\circ = 0.0618 + j0.6419$$

We shall now use the following two formulas for evaluating the hyperbolic forms

$$\cosh(\alpha + j\beta) = \cosh\alpha\cos\beta + j\sinh\alpha\sin\beta$$
$$\sinh(\alpha + j\beta) = \sinh\alpha\cos\beta + j\cosh\alpha\sin\beta$$

Application of the above two equations results in the following values

Cont.

$$\cosh\gamma l = 0.8025 + j0.037 \text{ and } \sinh\gamma l = 0.0495 + j0.5998$$

Therefore from (2.43) to (2.45) the ABCD parameters of the system can be written as

$$A = D = 0.8025 + j0.037$$

$$B = 43.4 + j240.72\,\Omega$$

$$C = -2.01\times 10^{-5} + j0.0015$$

ΔΔΔ

Example (2):

A single phase overhead transmission line delivers 1100 kW at 33 kV at 0·8 p.f. lagging. The total resistance and inductive reactance of the line are 10 Ω and 15 Ω respectively. Determine : (i) sending end voltage (ii) sending end power factor and (iii) transmission efficiency.

Solution.

Load power factor, $\cos\phi_R = 0{\cdot}8$ lagging

Total line impedance, $\vec{Z} = R + j\,X_L = 10 + j\,15$

Receiving end voltage, $V_R = 33$ kV $= 33{,}000$ V

$$\text{Line current, } I = \frac{kW \times 10^3}{V_R \cos\phi_R} = \frac{1100 \times 10^3}{33{,}000 \times 0{\cdot}8} = 41{\cdot}67 \text{ A}$$

$$\text{As } \cos\phi_R = 0{\cdot}8 \quad \therefore \quad \sin\phi_R = 0{\cdot}6$$

Cont.

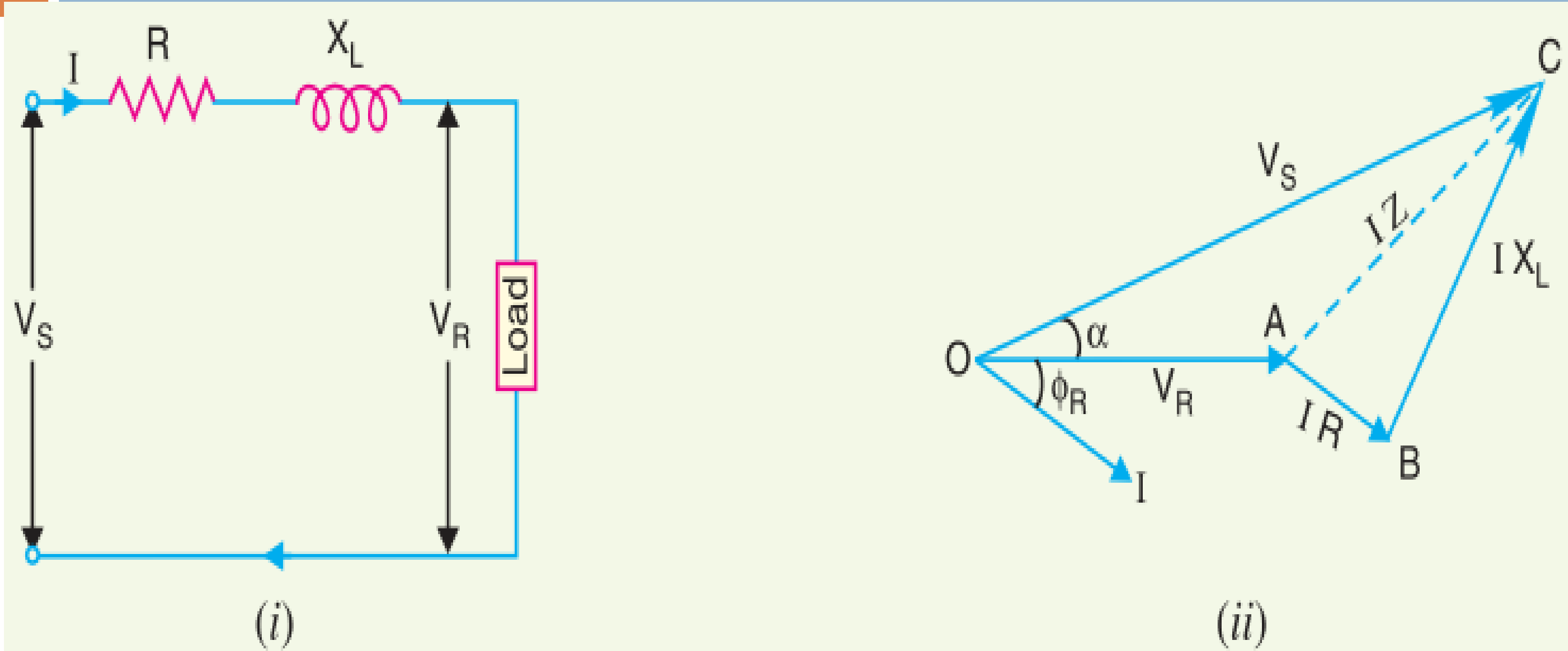

Fig. 10.5

The equivalent circuit and phasor diagram of the line are shown in Figs. 10.5 (*i*) and 10.5 (*ii*) respectively. Taking receiving end voltage $\vec{V}_R$ as the reference phasor,

Cont.

Performance of Single Phase Short Transmission Lines

$$\vec{V}_R = V_R + j\,0 = 33000 \text{ V}$$

$$\vec{I} = I(\cos\phi_R - j\sin\phi_R)$$

$$= 41{\cdot}67\,(0{\cdot}8 - j\,0{\cdot}6) = 33{\cdot}33 - j\,25$$

(*i*) Sending end voltage, $\vec{V}_S = \vec{V}_R + \vec{I}\,\vec{Z}$

$$= 33{,}000 + (33{\cdot}33 - j\,25{\cdot}0)\,(10 + j\,15)$$

$$= 33{,}000 + 333{\cdot}3 - j250 + j500 + 375$$

$$= 33{,}708{\cdot}3 + j\,250$$

$\therefore$ Magnitude of $V_S = \sqrt{(33{,}708\cdot3)^2 + (250)^2} =$ **33,709 V**

Cont.

(*ii*) Angle between $\vec{V_S}$ and $\vec{V_R}$ is

$$\alpha = \tan^{-1}\frac{250}{33{,}708\cdot 3} = \tan^{-1} 0{\cdot}0074 = 0{\cdot}42^{\circ}$$

∴ Sending end power factor angle is

$$\phi_S = \phi_R + \alpha = 36{\cdot}87^{\circ} + 0{\cdot}42^{\circ} = 37{\cdot}29^{\circ}$$

∴ Sending end p.f., $\cos\phi_S = \cos 37{\cdot}29^{\circ} =$ **0·7956 lagging**

(*iii*) Line losses $= I^2R = (41{\cdot}67)^2 \times 10 = 17{,}364 \text{ W} = 17{\cdot}364 \text{ kW}$

Output delivered $= 1100$ kW

Power sent $= 1100 + 17{\cdot}364 = 1117{\cdot}364$ kW

∴ Transmission efficiency $= \dfrac{\text{Power delivered}}{\text{Power sent}} \times 100 = \dfrac{1100}{1117\cdot 364} \times 100 =$ **98·44%**

Cont.

Note. V_S and ϕ_S can also be calculated as follows :

$$
\begin{aligned}
V_S &= V_R + I R \cos\phi_R + I X_L \sin\phi_R \text{ (approximately)} \\
&= 33{,}000 + 41{\cdot}67 \times 10 \times 0{\cdot}8 + 41{\cdot}67 \times 15 \times 0{\cdot}6 \\
&= 33{,}000 + 333{\cdot}36 + 375{\cdot}03 \\
&= 33708{\cdot}39 \text{ V which is approximately the same as above}
\end{aligned}
$$

$$
\begin{aligned}
\cos\phi_S &= \frac{V_R \cos\phi_R + I R}{V_S} = \frac{33{,}000 \times 0{\cdot}8 + 41{\cdot}67 \times 10}{33{,}708{\cdot}39} = \frac{26{,}816{\cdot}7}{33{,}708{\cdot}39} \\
&= 0{\cdot}7958
\end{aligned}
$$

Example (3):

A (medium) single phase transmission line 100 km long has the following constants :

Resistance/km = 0·25 Ω ; *Reactance/km = 0·8 Ω*

Susceptance/km = 14 × 10^{-6} siemen ; *Receiving end line voltage = 66,000 V*

Assuming that the total capacitance of the line is localised at the receiving end alone, determine (i) the sending end current (ii) the sending end voltage (iii) regulation and (iv) supply power factor. The line is delivering 15,000 kW at 0.8 power factor lagging. Draw the phasor diagram to illustrate your calculations.

Solution. Figs. 10.10 (*i*) and (*ii*) show the circuit diagram and phasor diagram of the line respectively.

Cont.

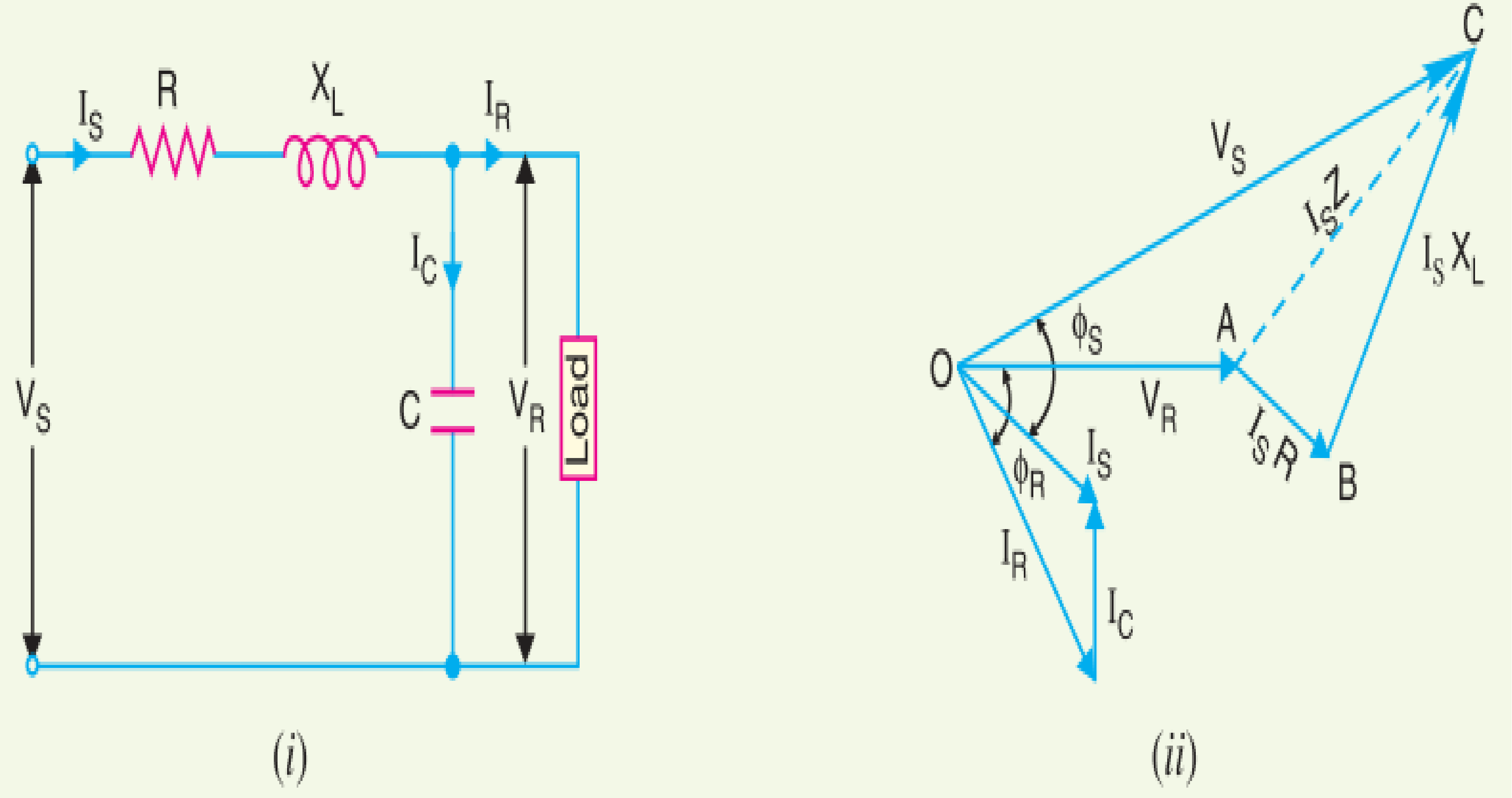

Fig. 10.10

Cont.

Total resistance, $R = 0{\cdot}25 \times 100 = 25\ \Omega$

Total reactance, $X_L = 0{\cdot}8 \times 100 = 80\ \Omega$

Total susceptance, $Y = 14 \times 10^{-6} \times 100 = 14 \times 10^{-4}\ S$

Receiving end voltage, $V_R = 66{,}000$ V

∴ Load current, $I_R = \dfrac{15{,}000 \times 10^3}{66{,}000 \times 0 \cdot 8} = 284$ A

$\cos\phi_R = 0{\cdot}8\ ; \qquad \sin\phi_R = 0{\cdot}6$

Taking receiving end voltage as the reference phasor [see Fig.10.10 (*ii*)], we have,

$$\vec{V_R} = V_R + j\,0 = 66{,}000V$$

Load current, $\vec{I_R} = I_R(\cos\phi_R - j\sin\phi_R) = 284\,(0{\cdot}8 - j\,0{\cdot}6) = 227 - j\,170$

Cont.

Capacitive current, $\vec{I_C} = jY \times V_R = j\,14 \times 10^{-4} \times 66000 = j\,92$

(*i*) Sending end current, $\vec{I_S} = \vec{I_R} + \vec{I_C} = (227 - j\,170) + j\,92$

$= 227 - j\,78$ ... (*i*)

Magnitude of $I_S = \sqrt{(227)^2 + (78)^2} =$ **240 A**

(*ii*) Voltage drop $= \vec{I_S}\,\vec{Z} = \vec{I_S}(R + j\,X_L) = (227 - j\,78)(\,25 + j\,80)$

$= 5{,}675 + j\,18{,}160 - j\,1950 + 6240$

$= 11{,}915 + j\,16{,}210$

Sending end voltage, $\vec{V_S} = \vec{V_R} + \vec{I_S}\,\vec{Z} = 66{,}000 + 11{,}915 + j\,16{,}210$

$= 77{,}915 + j\,16{,}210$ ...(*ii*)

Magnitude of $V_S = \sqrt{(77915)^2 + (16210)^2} =$ **79583V**

Cont.

(*iii*) % Voltage regulation $= \frac{V_S - V_R}{V_R} \times 100 = \frac{79{,}583 - 66{,}000}{66{,}000} \times 100 =$ **20·58%**

(*iv*) Referring to exp. (*i*), phase angle between $\vec{V}_R$ and $\vec{I}_S$ is :

$$\theta_1 = \tan^{-1} - 78/227 = \tan^{-1}(-0{\cdot}3436) = -18{\cdot}96^\circ$$

Referring to exp. (*ii*), phase angle between $\vec{V}_R$ and $\vec{V}_S$ is :

$$\theta_2 = \tan^{-1}\frac{16210}{77915} = \tan^{-1}(0{\cdot}2036) = 11{\cdot}50^\circ$$

$\therefore$ Supply power factor angle, $\phi_S = 18{\cdot}96^\circ + 11{\cdot}50^\circ = 30{\cdot}46^\circ$

$\therefore$ Supply p.f. $= \cos\phi_S = \cos 30{\cdot}46^\circ =$ **0·86 lag**

Example (4):

A 3-φ transmission line 200 km long has the following constants :

$$\text{Resistance/phase/km} = 0.16\ \Omega$$

$$\text{Reactance/phase/km} = 0.25\ \Omega$$

$$\text{Shunt admittance/phase/km} = 1.5 \times 10^{-6}\ S$$

Calculate by rigorous method the sending end voltage and current when the line is delivering a load of 20 MW at 0·8 p.f. lagging. The receiving end voltage is kept constant at 110 kV.

Solution :

Total resistance/phase, $R = 0.16 \times 200 = 32\ \Omega$

Total reactance/phase, $X_L = 0.25 \times 200 = 50\ \Omega$

Total shunt admittance/phase, $Y = j\,1.5 \times 10^{-6} \times 200 = 0.0003 \angle 90^\circ$

Series Impedance/phase, $Z = R + j\,X_L = 32 + j\,50 = 59.4 \angle 58^\circ$

The sending end voltage V_S per phase is given by :

$$V_S = V_R \cosh \sqrt{YZ} + I_R \sqrt{\frac{Z}{Y}} \sinh \sqrt{ZY} \qquad \ldots(i)$$

Now

$$\sqrt{ZY} = \sqrt{59\cdot4 \angle 58^\circ \times 0\cdot0003 \angle 90^\circ} = 0\cdot133 \angle 74^\circ$$

$$ZY = 0\cdot0178 \angle 148^\circ$$

$$Z^2Y^2 = 0\cdot00032 \angle 296^\circ$$

$$\sqrt{\frac{Z}{Y}} = \sqrt{\frac{59\cdot4 \angle 58^\circ}{0\cdot0003 \angle 90^\circ}} = 445 \angle -16^\circ$$

$$\sqrt{\frac{Y}{Z}} = \sqrt{\frac{0\cdot0003 \angle 90^\circ}{59\cdot4 \angle 58^\circ}} = 0\cdot00224 \angle 16^\circ$$

$$\cosh \sqrt{YZ} = 1 + \frac{ZY}{2} + \frac{Z^2 Y^2}{24} \text{ approximately}$$

$$= 1 + \frac{0\cdot0178}{2} \angle 148^\circ + \frac{0\cdot00032}{24} \angle 296^\circ$$

$$= 1 + 0\cdot0089 \angle 148^\circ + 0\cdot0000133 \angle 296^\circ$$

$$= 1 + 0\cdot0089\,(-0\cdot848 + j\,0\cdot529) + 0\cdot0000133\,(0\cdot438 - j\,0\cdot9)$$

$$= 0\cdot992 + j\,0\cdot00469 = 0\cdot992 \angle 0\cdot26^\circ$$

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Cont.

$$\sinh\sqrt{YZ} = \sqrt{YZ} + \frac{(YZ)^{3/2}}{6} \text{ approximately}$$

$$= 0\cdot133\angle\ 74° + \frac{0\cdot0024\angle\ 222°}{6}$$

$$= 0\cdot133\angle\ 74° + 0\cdot0004\angle\ 222°$$

$$= 0\cdot133\,(0\cdot275 + j\,0\cdot961) + 0\cdot0004\,(-\,0\cdot743 - j\,0\cdot67)$$

$$= 0\cdot0362 + j\,0\cdot1275 = 0\cdot1325\angle\ 74°6'$$

Receiving end voltage per phase is

$$V_R = 110\times10^3/\sqrt{3} = 63508\text{ V}$$

Receiving end current,

$$I_R = \frac{20\times10^6}{\sqrt{3}\times110\times10^3\times0\cdot8} = 131\ A$$

Cont.

Putting the various values in exp (*i*), we get,

$$V_S = 63508 \times 0{\cdot}992 \angle 0{\cdot}26^\circ + 131 \times 445 \angle -16^\circ 0' \times 0{\cdot}1325 \angle 74^\circ 6'$$

$$= 63000 \angle 0{\cdot}26^\circ + 7724 \angle 58^\circ 6'$$

$$= 63000\,(0{\cdot}999 + j\,0{\cdot}0045) + 7724\,(0{\cdot}5284 + j\,0{\cdot}8489)$$

$$= 67018 + j\,6840 = 67366 \angle 5^\circ 50' \text{ V}$$

Sending end line-to-line voltage $= 67366 \times \sqrt{3} = 116{\cdot}67 \times 10^3 \text{ V} =$ **116·67 kV**

The sending end current I_S is given by :

$$I_S = V_R \sqrt{\frac{Y}{Z}} \sinh \sqrt{Y Z} + I_R \cosh \sqrt{Y Z}$$

Putting the various values, we get,

$$I_S = 63508 \times 0{\cdot}00224 \angle 16^\circ \times 0{\cdot}1325 \angle 74^\circ 6' + 131 \times 0{\cdot}992 \angle 0{\cdot}26^\circ$$

$$= 18{\cdot}85 \angle 90^\circ 6' + 130 \angle 0{\cdot}26^\circ$$

$$= 18{\cdot}85\,(-0{\cdot}0017 + j\,0{\cdot}999) + 130\,(0{\cdot}999 + j\,0{\cdot}0045)$$

$$= 129{\cdot}83 + j\,19{\cdot}42 = 131{\cdot}1 \angle 8^\circ \text{ A}$$

∴ Sending end current = **131·1 A**